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Some aspects of creep rupture

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A bar, which is loaded in tension and is subject to creep, will rupture after a certain time. In some instances the rupture has a ductile character, in others it is brittle. It seems to be generally agreed that creep rupture is caused by the simultaneous action of two different mechanisms.

One cause of rupture is the steady decrease of the cross sectional area of the bar, due to the creep taking place at essentially constant volume. A phenomenological theory of creep rupture, based entirely on this phenomenon, was given by Hoff (1953). He also studied the formation of a neck, caused by some local deviation from the cylindrical shape of the bar.

Assuming the creep to be stationary throughout and governed by Norton's creep law

$$\frac{d\varepsilon}{dt}=\dot{\varepsilon}=k\sigma^n, \tag{1}$$

where ε = creep strain, σ = stress, t = time and k and n are constants, Hoff derived the critical time t^* after which purely ductile instability occurs under constant load

$$t^*=\frac{1}{nk\sigma_0^n}=\frac{1}{n\dot{\varepsilon}_0}. \tag{2}$$

The quantity t^* is defined as the time after which the smallest cross sectional area has decreased to zero and σ_0 denotes the magnitude of the stress at the instant of load application.

A graphic extrapolation method for determining t^* from a creep curve at constant load (and therefore increasing stress) based on eq. (2) is shown by Fig. 1.

The other cause of creep rupture is the damage caused to the material itself during creep. A phenomenological description of this process has been given by Kachanov (1958).

He introduced a creep damage parameter ψ, such that $\psi=1$ corresponds to the virgin state of the material and $\psi=0$ corresponds to a state of total creep damage, i.e. creep rupture. The hypothetical relation

$$\frac{d\psi}{dt}=-C\left(\frac{\sigma}{\psi}\right)^{\nu}, \tag{3}$$

where C and ν are constants,[1] was introduced, allowing a critical time t_K (sub-

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[1] Kachanov used the notations A and n instead of C and ν.

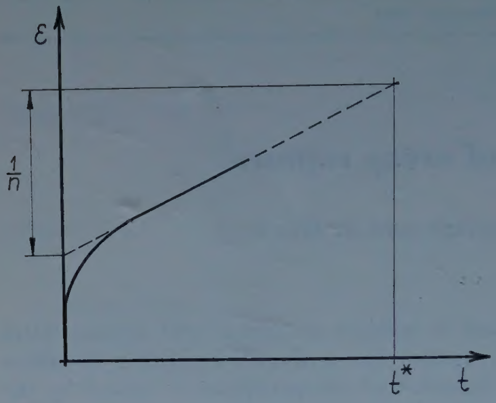

Fig. 1.

script K for Kachanov) to be determined, after which creep rupture occurs under constant stress. Kachanov was also able to determine the critical time t_K^* under constant load and therefore increasing stress, when both the ductility and the internal damage were taken into account.

It will be shown here that Kachanov's hypothesis (3) is nothing but an alternative formulation of the linear cumulative creep damage theory due to Robinson (1938).

The internal damage due to creep action can be interpreted as a decrease in the effective load carrying area of the bar. Such a decrease is caused, e.g., by the steady formation of micro-cracks and voids.

If A denotes the total cross sectional area of the bar and A_r denotes the real load carrying area, the damage caused by creep action can be defined as

$$D = \frac{A - A_r}{A}. \tag{4}$$

The corresponding stresses are σ and σ_r, defined by

$$\sigma A = \sigma_r A_r = P, \tag{5}$$

where P is the tensile load on the bar.

Assuming that the rate of damage is a function of the real stress σ_r only and not of the stress history, i.e.

$$\frac{dD}{dt} = f(\sigma_r), \tag{6}$$

there results in view of eqs. (4) and (5)

$$\frac{dD}{dt} = f\left(\frac{\sigma}{1 - D}\right). \tag{7}$$

In the special case, where f is a simple power function

$$f(x) = C x^{\nu}, \tag{8}$$

there follows with

$$1-D=\psi \tag{9}$$

the Kachanov hypothesis (3).

The differential equation obtained from eqs. (7) and (8) is separable and can be integrated. Considering the initial condition

$$D(0)=0 \tag{10}$$

and the creep rupture condition

$$D(t_R)=1, \tag{11}$$

where t_R is the creep rupture time, one obtains

$$C\int_0^{t_R}\sigma^\nu\,dt=\int_0^1(1-D)^\nu\,dD=\frac{1}{\nu+1}. \tag{12}$$

If σ were constant throughout the creep process, the corresponding creep rupture time t_K would be obtained from

$$C\,\sigma^\nu t_K=\frac{1}{\nu+1} \tag{13}$$

and hence eq. (12) can be given the alternative formulation

$$\int_0^{t_R}\frac{dt}{t_K}=1. \tag{14}$$

This is exactly the linear cumulative creep damage equation due to Robinson. It states that during each time increment the bar loses a certain fraction of its total creep lifetime, and that this fraction depends only on the duration of the time increment and on the current stress. Hence the interrelation between the hypotheses of Robinson and Kachanov has been demonstrated. Both are to be considered as special consequences of eq. (7) above.

If both the ductility of the bar and the internal creep damage are taken into account, there results the lifetime at constant load, cf. Kachanov

$$t_K^*=t^*\left[1-\left(1-\frac{n-\nu}{n}\cdot\frac{t_K}{t^*}\right)^{n/(n-\nu)}\right], \tag{15}$$

where t^* and t_K are given by eqs. (2) and (13) respectively. In deriving eq. (15) it was assumed that $\nu<n$.

It follows that $t_K^*<t^*$ only when $\sigma_0<\bar{\sigma}$, where σ_0 denotes the stress at the instant of load application and in particular

$$\bar{\sigma}=\left(\frac{1+\nu}{n-\nu}\cdot\frac{C}{k}\right)^{1/(n-\nu)} \tag{16}$$

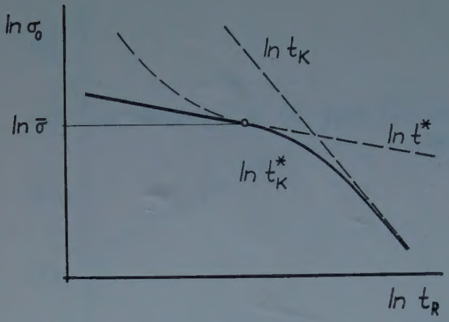

Fig. 2.

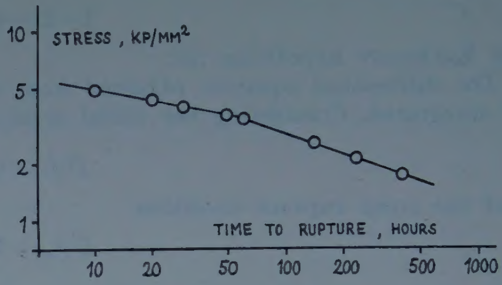

Fig. 3.

Hence the creep rupture will be caused by the simultaneous action of area reduction *and* internal damage if $\sigma_0 < \bar{\sigma}$ and by area reduction alone of $\sigma_0 > \bar{\sigma}$. Fig. 2 shows a doubly logarithmic plot of the creep lifetime versus the stress at the instant of load application. At stresses above $\bar{\sigma}$ the curve coincides with the straight line corresponding to eq. (2); at low stresses it approaches asymptotically the straight line corresponding to eq. (13).

Many experimental curves of creep lifetime versus stress show this appearance with two straight line portions, cf. Smith (1950) and Larson and Miller (1952). An example of the effect is shown in the diagram of Fig. 3, which concerns ingot iron subjected to creep rupture tests at 1100°F (593°C), cf. Thielemann and Parker (1939).

The magnitudes of n and ν can be read directly from the slopes of the two straight lines. One finds $n = 5.0$, $\nu = 2.7$ and hence the condition $\nu < n$ is fulfilled. The magnitude of n is representative of ingot iron at this temperature as known from ordinary creep tests.

It may be shown that eq. (8) is the only form of the function $f(x)$ which implies separability of the differential eq. (7) and hence the validity of Robinson's eq. (14).

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